

OC3140

Lab2 Probability Distribution

1. From the historical record, the January temperature at city A is normally distributed. The mean and standard deviation values are $22.2^{\circ}F$ and $4.4^{\circ}F$, respectively, use both standard normal table (Chapter3, P.23 figure 3.12) and matlab function normcdf.m answer the follow questions.
 - a. What is the probability that the temperature is colder than $21.4^{\circ}F$?
 - b. What is the probability that the temperature is between $22^{\circ}F$ to $23^{\circ}F$?
 - c. What is the probability that the temperature is hotter than $25^{\circ}F$?

Solution:

- a. $\mathbf{m}=22.2$, $\mathbf{s}=4.4$, $x=21.4$, $z=(x-\mathbf{m})/\mathbf{s}=(21.4-22.2)/4.4=-0.1818$, from the standard normal distribution table (Chapter III-23), we have

$$p(z > 0.18) = 0.4286, p(z > 0.19) = 0.4247.$$

Use the linear interpolation:

$$p(z > 0.1818) = \frac{(0.19 - 0.1818) * p(z > 0.18) + (0.1818 - 0.18) * p(z > 0.19)}{(0.19 - 0.18)} = 0.4279$$

$$p(z < -0.1818) = p(z > 0.1818) = 0.4279.$$

Use matlab function normcdf : $p(x < 21.4) = \text{normcdf}(21.4, 22.2, 4.4) = 0.4279$.

- b. $z_1 = \frac{22 - 22.2}{4.4} = -0.0455$, $z_2 = \frac{23 - 22.2}{4.4} = 0.1818$,

$$p(z < -0.0455) = p(z > 0.0455) = 0.45 \cdot p(z > 0.04) + 0.55 \cdot p(z > 0.05) = 0.4819,$$

$$p(z < 0.1818) = 1 - p(z > 0.1818) = 1 - 0.4279 = 0.5721,$$

$$p(-0.0455 < z < 0.1818) = p(z < 0.1818) - p(z < -0.0455) = 0.5721 - 0.4819 = 0.0902$$

Use matlab function: $p(22 < x < 23) = \text{normcdf}(23, 22.2, 4.4) - \text{normcdf}(22, 22.2, 4.4) = 0.0903$.

- c. $z = \frac{25 - 22.2}{4.4} = 0.6364$,
 $p(z > 0.6364) = 0.36 * p(z > 0.63) + 0.64 * p(z > 0.64) = 0.2623$,

Use matlab function: $p(x > 25) = 1 - p(x < 25) = 1 - \text{normcdf}(25, 22.2, 4.4) = 0.2623$

2. If another study found out that the January temperature at city A follows the Gamma distribution. Use matlab function gamcdf.m answer the same questions as problem-1.

Solution:

$$b = \frac{s^2}{m} = \frac{4.4^2}{22.2} = 0.8721, a = \left(\frac{m}{s}\right)^2 = 25.4566$$

- a. $p(x < 21.4) = \text{gamcdf}(21.4, 25.4566, 0.8721) = 0.4531$
 - b. $p(22 < x < 23) = \text{gamcdf}(23, 25.4566, 0.8721) - \text{gamcdf}(22, 25.4566, 0.8721) = 0.0888$
 - c. $p(x > 25) = 1 - p(x < 25) = 1 - \text{gamcdf}(25, 25.4566, 0.8721) = 0.2493$.
3. Use matlab function normpdf.m and gampdf.m plot and compare the PDF curves for Problems No.1 (normal distribution) and No.2 (Gamma distribution). Note: plot these two distributions on the same graph

Solution:

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>> mu=22.2; sigm=4.4; alf=25.4566; beta=0.8721;
>> x=mu+5*sigm*[-50:50]/50;
>> pnorm=pdf('norm',x,mu,sigm);
>> pgam=pdf('gam',x,alf,beta);
>> figure;
>> plot(x,pnorm); hold on;
>> plot(x,pgam,'r--');
>> ylim=get(gca,'ylim');
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>> plot(mu*[1,1],ylim,'.-');
>> text(mu-1.5,mean(ylim),["\mu = ",num2str(mu),
  '^oC'],'horizontalalignment','center','rotation',90);
>> legend('Normal Distribution','Gamma Distribution');
>> ylabel('Probability Density'); xlabel('Temperature (^oF)');

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